GENERALISED SOLUTION OF THE FIELD DUE TO A BURIED DIPOLE INSIDE A CONDUCTING MEDIUM BY USING T-O METHOD

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Abstract. In this work an attempt is made for the calculation of the electromagnetic field inside a conducting material by using the T-O method. As an excitation a current source is considered which is buried within the conducting medium. The T-O method provides flexibility in comparison with the classical method of the magnetic vector potential. For the numerical evaluation of integral equations of the method the boundary element method is used.

INTRODUCTION

The need for solving more complicated electromagnetic field problems gave rise to more sophisticated methods. The classical method in which the magnetic vector potential \( \mathbf{A} \) is used for solving three-dimensional problems is less attractive because the vector components are coupled and the magnetic and electric interface conditions are given with greater complexity and have to be computed throughout the region of interest. The method T-O which uses a generalized \( \mathbf{H} \) vector expressed by the electric vector potential \( \mathbf{T} \) and the scalar magnetic potential \( \mathbf{\Omega} \) is a flexible method controlling the coupled conditions [1].

In previous works [2], [3], [4] the problem of calculating the electromagnetic field due to current sources embedded in a conducting medium was examined for the case of horizontal or vertical dipoles. In this work the current dipole is placed in an arbitrary position and an attempt is made of giving a generalized solution for the determination of the field in this case. The problem is not limited to the power frequencies and it gives a description of the propagation phenomenon inside the conducting medium.

For the numerical treatment of the integral equations of the analytical formulation of the problem the boundary elements technique is applied [5].

DEVELOPMENT OF EQUATIONS

A current dipole is embedded in a conducting half-space medium at a distance \( h \) from the boundary and it is placed in an arbitrary position (Fig. 1).

Fig. 1. Geometrical configuration

The excitation is a current source, which diffuses a current density

\[
\mathbf{J}_s = I_0(r)e^{j\omega t} \mathbf{r}_0
\]

or

\[
\mathbf{J}_s = I_0(r)e^{j\omega t}(\sin\theta\cos\phi_r\sin\theta\sin\phi_i\cos\theta_r) \]

Considering the generalized \( \mathbf{H} \) as a combination of \( \mathbf{T} \) and \( \mathbf{\Omega} \), it is written

\[
\mathbf{H} = \mathbf{T} - \mathbf{\Omega}
\]

Reffering to the basic expression of the magnetic vector potential \( \mathbf{\nabla}\times\mathbf{A} = \mathbf{B} \), an analogous expression for the electric vector potential is written \( \mathbf{\nabla}\times\mathbf{T} = \mathbf{J} \) and the following equation are formed

\[
\nabla^2\mathbf{T} - \kappa^2\mathbf{T} = -\mathbf{J}_s
\]

\[
\nabla^2\mathbf{\Omega} = 0
\]

\[
\mathbf{\nabla}\mathbf{T} - \kappa^2\mathbf{\Omega} = 0
\]

where \( \kappa = j\omega\mu - \omega^2\epsilon \)

The use of a Green function method needs the definition of the equation

\[
\nabla^2\mathbf{\Omega} = -\delta(x'-x,y'-y,z'-z)
\]

where \( \delta = \frac{e^{-kr}}{4\pi r} \) the Green function for air,

\( r = [(x'-x)^2 + (y'-y)^2 + (z'-z)^2]^{1/2} \), (x,y,z) is the field point and (x',y',z') is the source point.

Applying Green's integral theorem, the above Helmholtz equations turn to the following integral equations
\[ R(x,y,z) = I(a \frac{\partial G}{\partial z}) \text{as } \text{az} (9) \]

Considering eqs. (2) and (8) the components of \( T \) on side 2 of the dividing surface \( ss' \) are written:

\[ T(x,y,z) = (\cos \theta \frac{\partial G}{\partial z} - \sin \theta \sin \phi \frac{\partial G}{\partial x}) - \int \left( T \frac{\partial G}{\partial z} - G \frac{\partial X}{\partial z} \right) ds \tag{10} \]

\[ T(x,y,z) = (\cos \phi \cos \theta - \sin \phi \sin \theta) - \int \left( T \frac{\partial G}{\partial x} - G \frac{\partial Y}{\partial x} \right) ds \tag{11} \]

\[ T(x,y,z) = (\sin \phi \cos \theta - \cos \phi \sin \theta) - \int \left( T \frac{\partial G}{\partial x} - G \frac{\partial Z}{\partial x} \right) ds \tag{12} \]

Eqs. (9), (10), (11) and (12) form a system of integral equations.

A similar system of integral equations is written for side 1 of the dividing surface \( ss' \), which belongs to the air.

The two sets of integral equations, which are formed in the two half-spaces 1 and 2 have the following 16 unknown quantities

\[ T_{k,i} (k=x,y,z \text{ and } i=1,2) \]

\[ \phi_{k,i} (i=1,2) \]

\[ \theta_{k,i} (i=1,2) \]

The above mentioned two sets of integral equations provide eight equations. Three equations are formed by the condition \( H = 0 \) for the half-space 1, in which \( H = 0 \) and one equation is given by using the assumption of the continuity of \( H \) on the surface \( ss' \), as an analog of the continuity of \( \phi \).

For the next two equations the boundary conditions on the surface \( ss' \) are applied. These conditions are given by the continuity of the tangential component of the vector \( \mathbf{H} \).

For the last two equations the gauge condition of eq. (6) and the continuity of the normal components of \( \mathbf{H} \) on the surface \( ss' \) are used.

**NUMERICAL TREATMENT**

For the numerical treatment of the analytical equations the boundary element technique is applied [5], [6]. According to the method the surface \( ss' \) is divided into subsections formed by concentric circles having as center the point \( 0' \) and radii starting also from \( 0' \). The following numerical equations are written.

\[ T_{k,i} = 2 \sum_{m} \left( T_{k,i} \frac{\partial G_{1,m}}{\partial z} - G_{1,m} \frac{\partial T_{k,i}}{\partial z} \right) ds \tag{13} \]

\[ \phi_{1,m} = -2 \sum_{m} \left( \phi_{1,m} \frac{\partial G_{1,m}}{\partial z} - \phi_{2,m} \frac{\partial G_{1,m}}{\partial z} \right) ds \tag{14} \]

\[ \theta_{2,m} = -2 \sum_{m} \left( \theta_{2,m} \frac{\partial G_{2,m}}{\partial z} - \theta_{1,m} \frac{\partial G_{2,m}}{\partial z} \right) ds \tag{15} \]

where \( k=x,y,z \), \( i=1,2 \) and \( m,l \) integers and

\[ L_{k,i} = 0 \tag{16} \]

\[ L_{k,i} = (\cos \phi \frac{\partial G}{\partial y} - \sin \phi \sin \theta \frac{\partial G}{\partial z}) \tag{17} \]

\[ L_{y,i} = (\cos \sin \phi \frac{\partial G}{\partial z} - \cos \theta \frac{\partial G}{\partial z}) \tag{18} \]

\[ L_{y,i} = (\sin \sin \phi \frac{\partial G}{\partial z} - \cos \theta \frac{\partial G}{\partial z}) \tag{19} \]

From eq. (13) and the conditions \( H_1 = H_2 \), \( T_1 = 0 \) and the continuity conditions as well as the assumption \( \phi_1 = \phi_2 \) on the surface \( ss' \), it is proved that \( T_{k} = T_{k}' = 0 \) on the surface \( ss' \).

After this result the quantities \( \phi_{x}, \phi_{y}, \theta_{x}, \theta_{y}, \theta_{z} \) are calculated. For the calculation of the remaining unknowns from eqs. (13) and (14) a matrix is fromed with (3m) elements. Using this matrix formulation the unknowns

\[ T_{k,i} \frac{\partial G_{1,m}}{\partial z}, \phi_{1,m} \frac{\partial G_{1,m}}{\partial z} \]

are calculated.

**NUMERICAL EXAMPLES**

For the evaluation of the numerical results a conducting medium with \( \sigma = 10^{-3} \Omega^{-1} \text{ m}^{-1} \) and \( \varepsilon_r = 1 \) is taken. The different quantities are computed by considering the frequencies \( f_1 = 50 \text{Hz} \) and \( f_2 = 50 \text{MHz} \). In Fig. 1 the frequency of the component \( H_x \) (versus \( x) \) is presented. In Fig. 2 the frequency of the component \( H_y \) (versus \( y) \) is presented. The component \( H_z \) is smaller than \( H_y \) because the orientation of the dipole favors the component \( H_y \).
CONCLUSION

The application of the T-Q method has the capability of solving 3-dimensional problems in a tractable way in comparison with classical analysis. The use of techniques for the numerical treatment of integral equations does not provide serious problems. The analyses of this work are extended to problems of propagation through media with limited conductivity. The possibility of strengthening a component depends on the apriori choice of the orientation of the dipole.