RIGOROUS ADI-FDTD ANALYSIS OF LEFT-HANDED METAMATERIALS IN OPTIMALLY-DESIGNED EMC APPLICATIONS

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Abstract – The precise modelling and broadband performance optimisation of 3-D EMC structures formed by composite left-handed metamaterials is presented in this paper through a new frequency-dependent ADI-FDTD method. Developing a class of multi-directional curvilinear schemes for these double-negative media, the unconditionally stable algorithm offers an advanced nodal control process which drastically suppresses the serious dispersion errors of existing approaches as time-step exceeds the Courant limit. Thus, the proposed framework leads to convergent discretisations that resolve all propagation bandwidths and enhance the design of promising periodic devices loaded by substrates of thin wires and split-ring resonators. Numerical results from realistic applications confirm the prior benefits along with the technique’s high accuracy and small computational overhead.

Introduction

One of the foremost scientific breakthroughs in modern electromagnetics is the conception and practical implementation of left-handed metamaterials (LHMs) [1], whose potential applicability has lately gained significant prominence. In an LHM, the concurrent tuning of both effective constitutive parameters to negative values results in a negative refractive index and other noteworthy properties not accessible in nature, such as inverted Snell law, Doppler shift or Cherenkov radiation. These features have been successfully engineered in the microwave regime [2-11] via thin metallic wires and split-ring resonators (SRRs). Nonetheless, since their electrical size is much smaller than a wavelength, any attempt to numerically explore LHM designs for efficient electromagnetic compatibility (EMC) components entails tools that should evade prolonged simulations. Pertinent to this requisite is the alternating-direction implicit finite-difference time-domain (ADI-FDTD) method which can theoretically skip stability conditions [12-14]. Extensive studies though, revealed that its reflection errors depend on mesh resolution and are critically amplified as time-steps become larger.

It is the objective of this paper, to introduce an enhanced frequency-dependent ADI-FDTD algorithm for the accurate analysis and wideband performance characterisation of composite LHM tailored to optimise 3-D EMC structures in arbitrary coordinate systems. The novel technique constructs a family of consistent spatial/temporal forms in order to reliably treat the demanding profile of the material’s feasible responses and fruitfully cope with the nontrivial backward-propagating fields localised to areas much smaller than the incident wavelength. Hence, the ensuing procedure provides robust discretisations for coarse lattices, while the generalised curvilinear formulation, combined with the ADI concepts, enables the rigorous approximation of the highly-varying permittivity and permeability functions. Furthermore, the irregular refractions at the LHM boundaries are carefully modelled through advanced stencil arrangements, volumetric nodal patterns and powerful perfectly matched layers (PMLs). In this manner, the intrinsic dispersion and anisotropy discrepancies are radically minimised even when temporal increments surpass the Courant criterion to a great extent. The aforementioned advantages are numerically validated by various up-to-date EMC devices like coupled SRR-loaded antennas or waveguides, high-pass filters for printed circuit boards (PCBs) and absorber linings in anechoic chambers that would otherwise necessitate lengthy time-domain investigations.

The Enhanced Frequency-Dependent ADI-FDTD Algorithm for 3-D LHM

A typical LHM is, in principal, synthesised by artificially-fabricated classes of inhomogeneities or small inclusions embedded in specific host media with the aim of achieving innovative and physically-realisable response functions, not encountered in naturally occurring materials. The most profitable implementation of these composite structures comprises periodic arrays of rectangular or circular SRRs in juxtaposition with
metallic rods or strip wires, as shown in Fig. 1. The SRR grid, which in the millimeter range exhibits a negative effective magnetic permeability, can be described by the Lorentz (LO) or Drude (DR) lossy model of

\[
\mu_{\text{eff}}^{\text{LO}}(\omega) = \mu_0 \left(1 - \frac{F \omega^2}{\omega^2 - \omega_0^2 + j\Gamma_0 \omega}\right), \quad \mu_{\text{eff}}^{\text{DR}}(\omega) = \mu_0 \left(1 - \frac{F \omega_0^2}{\omega(\omega - j\Gamma_0)}\right)
\]

where \(\omega_0\) and \(\Gamma_0\) are the analogous plasma and damping frequencies and \(\mu_0\) the static magnetic permeability.

On the other hand, the second LHM component, i.e. the network of thin wires, acts as a quasi-medium with a negative effective dielectric permittivity and can be, in the microwave spectrum, characterised by

\[
\varepsilon_{\text{eff}}^{\text{LO}}(\omega) = \varepsilon_0 \left[1 - \frac{(\varepsilon_s - 1)\omega^2}{\omega^2 - \omega_0^2 + j\Gamma_0 \omega}\right]
\]

and

\[
\varepsilon_{\text{eff}}^{\text{DR}}(\omega) = \varepsilon_0 \left[1 - \frac{(\varepsilon_s - 1)\omega_0^2}{\omega(\omega - j\Gamma_0)}\right]
\]

with \(\omega_0\) and \(\Gamma_0\) the relevant spatial increment.

Our major motive is the derivation of a basically dispersionless and unconditionally stable method competent of accurately handling the interspersed inclusions of an LHM in the time domain, without being affected by their small electrical size which leads to excessively long simulations. Therefore, we evaluate spatial derivatives through a numerically expedient and consistent set of \(K\)-th order operators given by

\[
L^k_\xi \left[ f_{\xi_{x,v,w}} \right] = \sum_{k=0}^{K} \frac{\left(\Delta \xi \right)^{2k+1}}{(3k + 2)!} A^k_{\xi} \left[ f_{\xi_{x,v,w}} \right] \sum_{n=0}^{K+1} q_n m^{2k}
\]

with

\[
A^k_{\xi} \left[ f_{\xi_{x,v,w}} \right] = \frac{\Lambda^k_{\xi}}{R \Delta} \sum_{n=-1}^{1} f_{\xi_{x,v,w}}^{\text{n}}
\]

where \(\xi\) belongs to the general curvilinear coordinate system \((\nu, v, w)\) and \(\Delta \xi\) is the relevant spatial increment. In (3), the multi-directional higher-order FDTD operator \(A^k_{\xi}[]\) launches adaptive stencil sets, \(q_m\), facilitating thus the study of complex material interfaces and permitting the extraction of abrupt fields, even of the backward-oriented wavefronts. Function \(Q_n\) and parameters \(q_m\) are adjustable degrees of freedom that treat outer boundaries and media discontinuities. Likewise, temporal integration is conducted in terms of

\[
T^k \left[ f_{\nu,v,w} \right] = \left( f_{\nu,v,w}^{t+\Delta t/2} - f_{\nu,v,w}^{t-\Delta t/2} \right) / Q_n (K \Delta t) - \partial_{\nu,v,w} \left[ f_{\nu,v,w} \right]
\]

and the suitable sinusoidal or exponential function \(Q_n\), whose selection can markedly improve the broadband performance of the algorithm, especially in the case of complicated LHM substrates, like the one of Fig. 2.
To develop the novel ADI-FDTD forms, magnetic \( \mathbf{B} = [B_x, B_y, B_z]^T \) and electric \( \mathbf{D} = [D_x, D_y, D_z]^T \) flux densities are denoted as
\[
\mathbf{B} = \mu_0 \mu_{\text{eff}}(\omega) \mathbf{H} - \mathbf{M}, \quad \mathbf{D} = \varepsilon_0 \varepsilon_{\text{eff}}(\omega) \mathbf{E} - \mathbf{P}, \quad (6),(7)
\]
where, for brevity, only the Lorentz representation of \( \mu_{\text{eff}} \) and \( \varepsilon_{\text{eff}} \) is depicted, with similar outcomes acquired by the Drude model. In (6) and (7), \( \mathbf{H} = [H_x, H_y, H_z]^T \), \( \mathbf{E} = [E_x, E_y, E_z]^T \) are the corresponding field intensities and \( \mathbf{M}, \mathbf{P} \) the magnetic and electric polarisations that incorporate frequency dependence. These auxiliary vectors satisfy
\[
\mathbf{M} = \frac{\mu_0 \mu_{\text{eff}}(\omega)^2}{\omega^2 - \omega_0^2 + j\Gamma_{\omega} \omega} \mathbf{H} \quad \text{and} \quad \mathbf{P} = \frac{\varepsilon_0 (\varepsilon_{\text{eff}} - 1) \omega_0^2}{\omega^2 - \omega_0^2 + j\Gamma_{\omega} \omega} \mathbf{E}. \quad (8),(9)
\]
Utilising (1)-(5) in Ampere’s and Faraday’s laws, one receives the following matrix equations dated
\[
Z^n L^K [\mathbf{H}] = T^K [\mathbf{D}] + \sigma \mathbf{E} + \mathbf{J} \quad \text{and} \quad Z^n L^K [\mathbf{E}] = -T^K [\mathbf{B}], \quad (10),(11)
\]
which combined with (6)-(9) give the complete Maxwell’s system. In (10), (11), operator \( L^K = [L_u, L_v, L_w] \), \( \sigma \) represents electric losses, \( \mathbf{J} = [J_u, J_v, J_w]^T \) describes external sources and \( Z^n, Z^L \) are diagonal media tensors.

The proposed technique circumvents the defects of usual configurations and splits each original iteration into two sub-iterations: the first one spans from \( n \) to \( n + 1/2 \) and the second from \( n + 1/2 \) to \( n + 1 \). Note that for symmetry, \( \mathbf{J} \) terms are replaced with temporal averages. Now, assume a dual-cell FDTD mesh and consider the update of \( E_u \). During the first sub-iteration, the \( u \)-direction, dispersion-optimised part of (10), yields
\[
4T^K \left[ D_u^{n+1/2}_{\text{l}1} + J_s^{n+1/2}_{\text{l}1} \right] + 2J_s^{n+1/2}_{\text{l}1} = Z^n L^K \left[ H_u^{n+1/2}_{\text{l}1} \right] - Z^n L^K \left[ H_u^{n+1/2}_{\text{l}2} \right] - \sigma \left( E_u^{n+1/2}_{\text{l}1} + E_u^{n+1/2}_{\text{l}2} \right) - 2J_s^{n+1/4}_{\text{l}1} + A_{\text{HO}}, \quad (12)
\]
with the subscript \( st1 = (i + 1/2, j, k) \). After the expansion of operator \( T[.] \) and some algebra, we derive
\[
D_u^{n+1/2}_{\text{l}1} - D_u^{n}_{\text{l}1} + Q_s(\Delta t) \left[ 4Z^n L^K \left[ H_u^{n+1/2}_{\text{l}1} \right] - 4Z^n L^K \left[ H_u^{n+1/2}_{\text{l}2} \right] - \sigma \left( E_u^{n+1/2}_{\text{l}1} + E_u^{n+1/2}_{\text{l}2} \right) - 2J_u^{n+1/4} + A_{\text{HO}} \right], \quad (13)
\]
where \( A_{\text{HO}} \) is a weighting function for all higher-order \( D_u \) temporal differentiations at \( n \) or earlier time-steps (known values). In (13), \( L_u[H_u] \) is implicitly computed by the unknown \( H_u \) values at \( n + 1/2 \), while \( L_u[H_s] \) is obtained via the already updated \( H_s \) at \( n \). To eliminate \( H_u \), the same ADI concept is employed in the \( w \)-directed part of (11), as
\[
T^K \left[ B_w^{n+1/2}_{\text{l}2} \right] = Z^n L^K \left[ E_u^{n+1/2}_{\text{l}2} \right] - Z^n L^K \left[ E_u^{n+1/2}_{\text{l}1} \right] \quad (14)
\]
for \( st2 = (i \pm 1/2, j + 1/2, k) \). Next \( B_u \) is inserted in (6) and the result is replaced in (13). Conversely, the unknown \( D_u \) component is calculated from (7) which, in its turn, requires the knowledge of \( P_u \). For the latter, used also in sub-wavelength waveguides (Fig. 3), apart from antennas, an unconditionally stable Crank-Nicolson scheme is applied in (9) to accomplish
\[
P_u^{n+1/2}_{\text{l}1} = \frac{4\varepsilon_0}{4\Gamma_x + Q_x(\Delta t)} \left( E_u^{n+1/2}_{\text{l}1} + E_u^{n+1/2}_{\text{l}2} \right) + \frac{4\Gamma_x - Q_x(\Delta t)}{4\Gamma_x + Q_x(\Delta t)} P_u^{n}_{\text{l}1} + \frac{\varepsilon_0}{4\Gamma_x + Q_x(\Delta t)} C_{\text{HO}}, \quad (15)
\]
with \( C_{\text{HO}} \) the corresponding weighting term that collects all higher-order temporal derivatives. Substitution of (14) and (15) into (13) for each \( j \) along \( v \) axis, produces the sparse, three-band, tridiagonal system of
\[
\begin{aligned}
\partial_j E_u^{n+1/2}_{\text{l}1, j+1/2, k} - \partial_j E_u^{n+1/2}_{\text{l}1, j-1/2, k} = & \partial_j E_u^{n}_{\text{l}1, j+1/2, k} + \partial_j E_u^{n+1/2}_{\text{l}1, j-1/2, k} + \partial_j L^K \left[ H_u^{n+1/2}_{\text{l}1, j+1/2, k} \right] \\
- \partial_j L^K \left[ H_u^{n+1/2}_{\text{l}2, j+1/2, k} \right] - \partial_j \left( L^K \left[ E_u^{n}_{\text{l}1, j+1/2, k} \right] + L^K \left[ E_u^{n+1/2}_{\text{l}1, j+1/2, k} \right] \right) + \partial_j P_u^{n+1/2}_{\text{l}2, j+1/2, k}
\end{aligned}
\]
which can be recursively solved. Parameters \( \partial_j \) \( (m = 1,2,\ldots,9) \) are defined by the spatial stencils and functions \( Q_x, C_{\text{HO}} \). In an akin way, the second sub-iteration for \( E_u \) reverses the update of \( L_u[H_u] \) and \( L_u[H_s] \) in the interval \( n + 1/2 \) to \( n + 1 \). Once all quantities are computed, the technique proceeds to the next time-step.
Stability and Dispersion Considerations

To delve into the stability of our algorithm, the von Neumann method expresses the two sub-iterations as

\[
\begin{align*}
[n, n+1/2]: & \quad \Xi_2 E^{n+1/2} = \Theta_1 E^n \\
[n+1/2, n+1]: & \quad \Xi_2 E^{n+1} = \Theta_1 E^{n+1/2} \quad \Rightarrow E^{n+1} = \Xi_2 \Theta_1 \Xi_2^{-1} \Theta_1 E^n = \Psi E^n.
\end{align*}
\]

Sparse matrices $\Xi_2, \Theta_1$ (for $l = 1, 2$) are generated through the suitable material parameters appearing in the final system of equations, e.g. (16), while $\Xi_2^{-1}$ must be evaluated only for the nodes at the left-most lattice area, since the electric field at the remaining points is efficiently calculated via the other two neighbouring cells. Subsequently, analysis continues with the solution of the related eigenvalue problem to provide

\[
\lambda_{1,2} = 1 \quad \text{and} \quad \lambda_s = \sqrt{2 R_1^2 - 3 R_2^2 \pm j(R_s / 2 - R_t)} / R_t \quad \text{for} \quad s = 3, \ldots, 6, \quad (18)
\]

in which $R_1, R_2$ are functions of $\mathcal{K}_\zeta = (\Delta t/\Delta \zeta) \sin(k_\zeta \Delta \zeta/2)$ for $\zeta \in (u,v,w)$, $k_\zeta$ is the wavenumber and $j^2 = -1$. Evidently, the magnitudes of all eigenvalues in (18) are less or equal to unity, a deduction that substantiates the unconditional stability of the formulation. This considerable property enhances simulation convergence and subdued reflection errors independently of mesh resolution unlike most of the regular approaches.

Finally, the dispersion relation is extracted by defining the associated error, $e_{\text{disp}}$, in terms of the numerical $k_{\text{num}}$ and the exact $k_{\text{ex}}$ wavenumbers. In particular, the ADI-FDTD solution is presumed to be the superposition of propagating and evanescent waves, enabling thus a far more realistic and precise investigation of every inherent mechanism. After the appropriate mathematical manipulations, this process leads to

\[
k_{\text{num}}(\omega) \equiv \omega \left(1 - \frac{5(\Delta t)^2 K_{2k+1}}{749} + O((\Delta t)^{K_{2k+1}})\right), \quad \text{and} \quad e_{\text{disp}} \equiv \frac{2 + 5(\Delta t)^2 K_{2k+1}}{749} \omega. \quad (19),(20)
\]

Equation (19) exhibits a powerful enhancement – when compared to the exact relation $k_{\text{ex}}(\omega) = \omega$ or the FDTD one – easily adjusted by the accuracy order $K$, without increasing CPU or memory burden. As a consequence, dispersion error (20) is strongly suppressed and the most important: its minimisation is more prominent (up to 7 or 8 orders of magnitude) when time-steps significantly exceed the Courant criterion.

Numerical Results

The enhanced technique, for $K = 3$ and $5$, is applied to various 3-D EMC structures optimised by specific LHM designs. Simulations involve both the frequency-dependent models and the equivalent thin wire/SRR arrays. Due to the merits of (3)-(5), grid resolution receives fairly coarse values (e.g. $R = \lambda/7$), while diverse excitations and modified PMLs complete each numerical configuration. We, first, study a $5 \times 5$ planar patch antenna fixed on the properly-tuned LHM substrate of Fig. 4, which is loaded either by rectangular ($l_r = 3.1$ mm, $r = 0.96$ mm, $l_g = 0.35$ mm, $g = 0.31$ mm, $d = 0.28$ mm) or circular ($l_r = 3.04$ mm, $r = 0.94$ mm, $l_g = 0.32$ mm, $g = 0.3$ mm, $d = 0.28$ mm) SRRs. Fig. 6a shows the $H_v$ variation along a transverse substrate cut and Fig. 6b the large reduction of the maximum dispersion error vs CFLN = $\Delta \text{Prop} / \Delta t_{\text{ADITD}}$ ratio, even in the case of small $R$. Next, a 16-layer LHM – comprising circular SRRs with $l_g = 3$ mm, $r = 0.8$ mm, $g = 0.36$ mm, $d = 0.34$ mm, as in Fig. 1b – constructs the substrate of a $7 \times 7$ smart antenna array. The domain is discretised in $48 \times 56 \times 32$ cells with $\Delta t_{\text{Prop}} = 0.8267$ ns rather than the $162 \times 174 \times 138$ cells and $\Delta t_{\text{ADITD}} = 0.041$ ns of the typical ADI-FDTD method. The frequency gap in the structure’s transmitted power is shown in Fig. 6c, where the serious improvement achieved by the LHM substrate is, easily, detected.

The third application deals with the behaviour of a high-pass PCB filter realised via the spiral inductor-SRR element (Fig. 1c) according to the pattern of Fig. 2. Its dimensions are $l_\lambda = 9.4$ mm, $l_a = 9.2$ mm, $l_c = 5.8$ mm, $l_e = 6$ mm, $l_g = 0.4$ mm, $w = 0.2$ mm, $d = 0.2$ mm

Fig.4 Part of the LHM substrate of a planar antenna

Fig.5 A hybrid urethane absorber backed by an LHM
and $l_m = 18.6$ mm. The proposed algorithm involves a $62 \times 54 \times 30$ lattice which is virtually 90\% coarser than the conventional one and uses a temporal increment 25 times beyond the Courant limit. Fig. 6d gives the characteristic impedance of the filter for a $6 \times 6$ and $9 \times 9$ arrangement, while Fig. 6e shows its return loss for a specific frequency range. As deduced, our formulation is superior to the second-order ADI FDTD scheme from an accuracy and convergence viewpoint, since its results are very close to the reference solutions.

To proceed with the validation, we focus on the optimisation of sub-wavelength waveguides – terminated by a 6-cell PML – containing slabs of transversely- or longitudinally-located networks of thin wires and circular SRRs (Fig. 3) like those of the first example. In this context, Table I provides the first resonance frequency, the solution and the max dispersion error for a transversely SRR-loaded waveguide ($l_x = 5.2$ mm, $l_y = 26.2$ mm, $l_z = 1.8$ mm) regarding three $l_m$ values. Notice the high precision and grid decrease attained by the new scheme, despite the large CFLN values. Similar observations are obtained from Fig. 7 which illustrates the $E_z$ variation at the waveguide end. Conversely, Fig. 8 gives the $S_{11}$ and $S_{12}$ parameters for a longitudinally SRR-loaded structure, where our technique ($25 \Delta t$, $\Delta t = 1.534$ μs, grid: $68 \times 126 \times 32$, time-steps: 2524) outperforms all other approaches ($1.476 \Delta t$, $\Delta t = 0.986$ ns, grid: $136 \times 262 \times 68$, time-steps: 45432).

The performance of a hybrid absorber lining, backed by an LHM (Fig. 5), in a $10.8 \times 7.2 \times 11.4$ m anechoic EMC chamber is, now, analysed. This electrically-large structure is discretised in $92 \times 84 \times 76$ cells with $\Delta t_{Prop} = 0.231$ ns. Fig. 9 presents the reflectivity of different absorbers with the proposed one achieving the best results. Moreover, we explore the influence of the gap between the ferrite tiles. Thus in Fig. 10, electric field variation is computed in a region 1.2 m above the turntable for a 1.45 mm and a 0.20 mm gap. Although the difference is small, the first environment is totally unreliable for any EMC test in contrast to the second one which yields an unperturbed quiet zone devoid of spurious modes. Finally, Fig. 11 shows the chamber’s field uniformity. The agreement of the enhanced procedure (3685 time-steps) with the reference data is very sufficient, unlike that of Yee’s scheme for a 89\% denser mesh, $\Delta t_{FDTD} = 0.053$ ns and 81070 time-steps.

Table I First resonance frequency for different configurations of the transversely SRR-loaded waveguide

<table>
<thead>
<tr>
<th>Ref. (GHz)</th>
<th>$l_m$ (mm)</th>
<th>ADI-FDTD</th>
<th>CFLN</th>
<th>Sol. (GHz)</th>
<th>Error (%)</th>
<th>Lattice</th>
<th>Reduction (%)</th>
<th>Max dispersion</th>
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<td>6.8637</td>
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<td>6.6228</td>
<td>3.5097</td>
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<td>0.0116</td>
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<td>7.6731</td>
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<tr>
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<td>0.0321</td>
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**Conclusions**

The potential of designing optimal EMC devices composed of tuneable LHM is presented in this paper via a rigorous 3-D ADI-FDTD method. Implementing a flexible discretisation concept, the proposed formulation minimises dispersion errors beyond the Courant stability condition. Numerical results prove these enhancements, suggesting thus a proficient means for the broadband analysis of ultra-small configurations.

**References**


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