A topologically consistent class of 3-D higher order curvilinear FDTD schemes for dispersion-optimized EMC and material modeling

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Abstract
An enhanced higher order finite-difference time-domain (FDTD) algorithm for the precise analysis of complex 3-D electromagnetic compatibility (EMC) structures and arbitrary interface material distributions in general curvilinear, skewed and stretching lattices is presented in this paper. Introducing a systematic topological tessellation, the novel methodology develops a family of robust higher order non-standard forms, which exactly represent the material properties and significantly suppress the artificial dispersion errors. To handle inherent mesh deficiencies an enhanced low-pass filtering procedure is implemented, while a consistent class of self-adaptive compact operators ensures the correct evaluation of electromagnetic components near boundary walls. Moreover, for more involved media interfaces that do not follow the lines of the grid, a convergent transformation around these discontinuities leads to precise simulations. Therefore, this optimal field-preserving technique along with suitably tuned perfectly matched layers (PMLs), achieve high levels of accuracy, decrease the required number of points per wavelength and provide considerable computational savings, as indicated by extensive numerical results addressing 3-D EMC and hard-to-model material applications.

Keywords: Material modeling; Computational electromagnetics; Finite-difference time-domain methods; Higher order schemes; General curvilinear coordinates; Numerical solution of partial differential equations

1. Introduction
The progressively increasing demands for advanced simulations of electromagnetic compatibility (EMC) devices and the reliable materials processing of their complicated components have lately mandated strict stipulations on modeling and design tools [1–3]. Among existing numerical techniques, the finite-difference time-domain (FDTD) method has a wide applicability in many areas of research [4]. The original algorithm is relatively simple in the construction of the computational lattice, since no mathematically involved formulae are required. However, when solving an arbitrarily-curved problem via a second-order staircase discipline like the FDTD one, the artificial dispersion mechanisms place a major restraint to its usage. Also, several strenuous subwavelength geometries and multi-dimensional material compositions often prohibit the reliable interpretation of the underlying physics.

Not to mention that when a field component is discontinuous along a curvilinear interface, the FDTD scheme may exhibit loss of global convergence and stability. Actually, the prior drawbacks have been the subject of an ongoing research [5–10], aiming at approaches that offer acceptable stability.

In this paper, a multi-space 3-D higher order FDTD methodology—based on generalized non-standard curvilinear operators—is presented for the systematic investigation of EMC and complex media structures. Using the language of algebraic topology, the unified framework hosts a covariant/contravariant flux theory which along with an appropriately established dual-grid formulation enables the correct mapping of acutely deformed meshes. This strategy, by means of a low-pass filtering process, optimizes the overall performance and subdues dispersion errors. Moreover, the resulting higher order FDTD expressions take into account a sufficiently larger amount of points for spatial derivatives, instead of the two points employed by the traditional counterparts. To overcome the inevitably widened spatial stencils...
near absorbing or perfectly electric conducting (PEC) boundaries, we develop a family of curvilinear compact differencing schemes, while all infinite spaces are terminated by a modified version of the efficient perfectly matched layer (PML) \[11,12\]. For the important case of dissimilar materials, whose interface does not coincide with any of the grid axes, the algorithm introduces a special transformation combining additional degrees of freedom. This perspective guarantees the fulfilment of the proper jump-conditions and the calculation of precise values for the corresponding constitutive parameters. Numerical results, studying a variety of 3-D curvilinear EMC applications and problems with diverse materials, depict the merits of the proposed method, even in coarse FDTD resolutions or geometric singularities.

2. The generalized higher order curvilinear FDTD algorithm

When objects of arbitrary curvature and multiple materials are to be modeled, the FDTD technique exhibits an ill-suited behavior, mainly, due to the incomplete imposition of continuity conditions at the interfaces. Unfortunately, to these errors one must add the lattice dispersion discrepancies in intrinsic in the low-order schemes. A possible solution could be the averaging of constitutive parameters at the interfaces. However, in the case of ambiguous regions such a procedure is not at all exact or convergent. Actually, the above serious defects have been the primary motive for the development of our method, especially in curvilinear grids.

2.1. Construction of the consistent non-standard forms

The essential premise of the higher order (HO) algorithm resides in the representation of electromagnetic fields via a new class of 3-D non-standard forms. Their form is given by

\[ W^{\text{HO}}_{q,L}[f'_{u,v,w}]=\frac{R_q}{C_L(L,k_0)} \sum_{n=1}^{M} R^{\text{ho}}_{n} \times \sum_{l=1}^{N} f^{l}_{u,v,w} D^{\text{ho}}_{q,L}[f'_{u,v,w}], \]  

(1)

where \( M \) is the order of accuracy and \( q \) is a variable of the general coordinate system \( (u, v, w) \) described by its respective \( q \) metrics. Such a differencing rationale is characterised by dispersion error mechanisms, having the potential to spoil the final outcomes, are drastically subdued or even completely eliminated. On the other hand, parameters \( R^{\text{ho}}_{n} \) and \( f^{l}_{u,v,w} \) achieve an inherent robustness both in the handling of geometric details and the right assignment of field quantities to space-time entities by satisfying the following gauges

\[ \sum_{n=1}^{M} R^{\text{ho}}_{n} = 1. \]  

(2)

Factor \( L \) defines the number of stencils, \( lbq \), along each axis, which are needed for the accomplishment of a specific accuracy with a typical value of \( L = 3 \). The correction function \( C_{q}(k_0lbq) \) ensures the smooth transition from the continuous to the discrete state. Its argument, depending on wavenumber \( k \), is selected to handle broadband electromagnetic excitations with the non-standard concepts. This is conducted by employing the Fourier transform of the already computed electric or magnetic vectors at predetermined lattice positions. The process does not affect the total overhead, whereas its efficacy increases with the number of prefixed nodes. A possible choice of \( C_{q} \) could be

\[ C_{q}(k_0lbq) = \frac{16}{\pi} \sin \left( \frac{k_0lbq}{2} \right) \cos \left( \frac{k_0lbq}{2} \right). \]  

(4)

Operators \( D^{\text{ho}}_{q,L}[f'_{u,v,w}] \), in (1), cover all optimal node arrangements – irrespective of cell shape – leading to very coarse grids and mutual cancellation of material discrepancies. Therefore, unlike the common FDTD technique which involves only two mesh points for the calculation of spatial derivatives, the proposed method concerns a whole set of nodes. This remark is crucial for body-fitted tessellations with abrupt geometric attributes like slope discontinuities, skewness or stretching. An indicative \( v \)-directed \( D^{\text{ho}}_{v,L}[f'_{u,v,w}] \) is

\[ D^{\text{ho}}_{v,L}[f'_{u,v,w}] = \frac{\partial}{\partial v} \left[ f'_{u,v,w} + \sum_{\gamma \neq v \pm 1} f'_{u,v+w,\gamma} \right]. \]  

(5)

Analogous expressions can be extracted towards the remaining \( u \) and \( w \) directions in the domain.

The aforementioned HO non-standard operators are now applied to the discretization of spatial and temporal derivatives appearing in Maxwell’s curl equations. In this framework, the generalized forms are established via a series of parametric relations, depending on the approximation level. Hence,

\text{spatial differentiator:}

\[ \mathbf{L}_{q}[f'_{u,v,w}] = \frac{b_1}{k_0} \left[ W^{\text{ho}}_{q,L}[f'_{u,v,w}] + \sum_{l=1}^{N} f^{l}_{u,v+w} \right], \]  

(6)

\text{temporal differentiator:}

\[ \mathbf{T}_{l}[f'_{u,v,w}] = \frac{1}{C_T(\delta t)} \left( f'_{u,v+w} - f'_{u,v-w} - b_2 \partial_t f'_{u,v,w} \right), \]  

(7)

with \( b_1 \) and \( b_2 \) being specific tuning parameters and \( C_T(\delta t) \) the correction function of \( \mathbf{T}_{l} \). It stressed that (6) is applied everywhere in the domain except absorbing or PEC walls,
The new scheme utilizes the idea of fluxes for the fields where its stencils extend at least two nodes on either side of secondary lattice. The non-orthogonal dual-grid formulation

Let us recall the previously defined coordinate system \((u = \text{Ehx}, v = \text{jhy}, w = \text{khu})\) and divide a given 3-D domain into uniform cells, as in Fig. 1. The center of every primary cell is positioned at \((i,j,k)\), while secondary ones are centered on the vertices of the primary grid. This simply implies that covariant, \(h_q\), and contravariant, \(h^p\), components \((p,q = u,v,w)\) of magnetic vectors, \(\mathbf{H}\), are located at primary face centers remaining so in complete interleaving with \(e_q\) and \(e^p\) quantities of electric vectors, \(\mathbf{E}\), placed at edge centers. The new scheme utilizes the idea of fluxes for the fields across the faces defined by \(\delta^p = g^1/2 f^p(t) f = e, h\), with \(g_{qp}\) the coordinate metrics. Representation of \(f_q^p\) with \(f^p\) is conducted by the chain rules \(f_q^p = g_{qp}f^p\) and \(\delta^p = g_{qp}f_q^p\). Next, we introduce the linear operator \(Q^{(\alpha)}\) such that \(f^{p+1} = \mathbf{Q}^{(\alpha)}f^p\), which uses the local components \(f_q^p\) and the neighboring ones \(f_{q+1}^p, f_{q+2}^p (q + 1, q + 2\) denote a consecutive cyclic permutation of \(u, v, w\), multiplied by the discrete metric terms. For instance, the \(Q^{(\alpha)}[h_q]\), along \(u\) axis, is written as

\[
Q^{(\alpha)}[h_q^{i+1/2, j, k}] = \partial^\alpha h_q^{i+1/2, j, k} \nonumber
\]

\[
= \frac{1}{2} \left[\sum_{q=1}^{1} \partial^{\mu x} (h_q^{i+1/2, j, k} h_q^{i+1/2, j, k}) + \partial^{\mu x} (h_q^{i-1/2, j, k} h_q^{i-1/2, j, k}) + \partial^{\mu x} (h_q^{i+1/2, j, k} h_q^{i-1/2, j, k}) + \partial^{\mu x} (h_q^{i-1/2, j, k} h_q^{i+1/2, j, k})\right],
\]

where \(\partial^\alpha\) is the vector of spatial operators.\

\[
E^{(\alpha)} = \frac{1}{2} \left(\sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k})\right),
\]

which, due to their ability to follow boundary curvatures, are designated as self-adaptive. Having completed the theoretical configuration of the proposed procedure, we will now continue with the construction of the topologically robust dual meshes that discretize the continuous physical space.

2.2. The non-orthogonal dual-grid formulation

where \(Q^{\alpha} = g^{\alpha/2} \delta^{\alpha}\). Flux analysis is very convenient since it avoids the complicated projection forms for the dually-staggered fields and provides an effective representation of dissimilar material interfaces, since it always focuses on the rigorous evaluation of the constitutive parameters without entailing any constraints. Through the prior process and the substitution of curl operators, Ampere’s and Faraday’s laws become

\[
\mathbf{E}^{\alpha} = \frac{1}{2} \left(\sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k})\right),
\]

in which \(L = [L_u, L_v, L_w]\) is the vector of spatial operators. Also, \(E_{\alpha} = [e_u, e_v, e_w]\), \(H_{\alpha} = [h_u, h_v, h_w]\) are matrices of the unknown covariant components, \(J = \sigma \mathbf{E}, \mathbf{M} = \sigma \mathbf{H}\) the corresponding conduction current densities. \(\sigma, \mathbf{R}, \mathbf{C}\) constitutive matrices defining every material interface and \(G^u, G^v\) the suitable metric tensors, respectively. Matrices \(T_{\alpha} = \partial^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k}) + \sum_{p=1}^{3} \epsilon^{ij} \delta^{\mu x} (E_i^{p+1/2, j, k} E_j^{p+1/2, j, k})\right),
\]

(11)

The major aspect of stability for (6) and (7) is extracted by the von Neumann analysis, which yields

\[
clt \leq \left(\sum_{q=1}^{n} \sum_{p=1}^{n} g^{\alpha p} \right)^{-1/2}.
\]

As expected, the dispersion relation is remarkably improved due to the scalable accuracy coefficients \(M\) and \(L\) of (1). The optimization level may be directly experienced by comparing the dispersion relation of the HO non-standard FDTD algorithm with that of the second-order Yee scheme. So, after
In fact, it is the careful choice of parameters $R_q$ that main receives the following form of control of filtering, the expression in the interior of the do-

$$S_q = \left( \frac{dt}{\delta q} \right) \sin \left( \frac{k q \delta q}{2} \right),$$  \tag{13}$$

where one can easily observe that (13) is a very small fraction of the usual FDTD relation $P_q(t)$. As an illustration, we provide the dispersion relations for the two methods, for $M = 4$, $L = 3$, $c'$, the numerical phase velocity, $\beta = 2\pi/(k \delta q)$ and $\delta$ the incident angle. Hence,

$$\left( \frac{c}{\beta} \right)^{-\sin} \approx 1 - \frac{\pi^2}{8} + \frac{1}{12} \cos(4\delta),$$ \tag{14}$$

$$\left( \frac{c}{\beta} \right)^{\cos} \approx 1 - \frac{5\pi^2}{19} + \frac{1}{12} \cos(4\delta).$$ \tag{15}$$

In fact, it is the careful choice of parameters $R_q$ and $P_{1,2}$ that leads to this serious improvement. Overall, the proposed HO methodology creates a set of topologically robust forms, which, by constructing a more consistent cell, diminishes dispersion errors and avoids conformal techniques. For the termination of infinite problems, we launch a curvilinear non-standard version of the PML, based on an unsplit-field realization. The optimized absorber is built using the proper scaling that retains the original field variation and incorporates the suitable coordinate transformation.

2.3. The low-pass spatial filtering procedure

A critical factor that may have a destabilizing effect on HO simulations is the non-uniform nature of the FDTD grid, created by the arbitrary alignment of material interfaces. One viable approach stems from the annihilation of spurious high frequencies, which are likely to be aroused by the numerical method in the vicinity of the media boundaries. Herein, we employ a post-processing algorithm applied to the cell average $G_{\delta q}$

$$G_{\delta q} = \frac{1}{N_q} \sum_{i=0}^{N_q} G_{\delta q,i},$$ \tag{16}$$

with the limits of integration $[q_1, q_2]$ being the dimensions $[u_1, u_2], [v_1, v_2], [w_1, w_2]$ of the cell edges and $G$ any component of vectors $E_{\delta q}, H_{\delta q}$. In the case of multi-space problems, (16) is used sequentially along the directions $u,v,w$ of the lattice. Introducing an additional degree of freedom, $\Xi$, for the control of filtering, the expression in the interior of the domain receives the following form

$$G^* = \frac{1}{2} \sum_{i=0}^{N_q} G_{\delta q,i}$$

$$\Xi = \frac{1}{2} \sum_{i=0}^{N_q} G_{\delta q,i} + \epsilon_{\delta q,i} + \eta_{\delta q,i} \epsilon_{\delta q,i}$$ \tag{17}$$

where the primed quantities represent the filtered field values. The frequency response of (17) is

$$FS(k) = \sum_{i=0}^{N_q} \epsilon_{\delta q,i} \cos(k \delta q)$$ \tag{18}$$

which has $\Xi = 2$ unknowns. Owing to the symmetric profile of (17), $FS(k)$ is real and thus the filter acts only on the magnitude of every field quantity. To suppress the highest frequency mode, we require that $FS(k) = 0$ and obtain $\Xi + 1$ equations by matching the appropriate Taylor series expanded about $k$. A frequently selected value of $\Xi$ is four with $\epsilon_{\delta q,i} = 0.3514, a_{i} = 1.2875874935, a_{i} = -1.927896463, a_{i} = 2.5751793249$ and $a_{i} = 0.3468213796$, while a complete set of filtering coefficients for various approximation orders is given in Table 1.

Table 1 Filtering coefficients of diverse approximation orders and stencil sizes

<table>
<thead>
<tr>
<th>Approximation order</th>
<th>Filtering coefficients</th>
<th>Stencil size</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$a_{0}, a_{1}$</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>$a_{0}, a_{1}, a_{2}$</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>$a_{0}, a_{1}, a_{2}, a_{3}$</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>$a_{0}, a_{1}, a_{2}, a_{3}, a_{4}$</td>
<td>9</td>
</tr>
<tr>
<td>7</td>
<td>$a_{0}, a_{1}, a_{2}, a_{3}, a_{4}, a_{5}$</td>
<td>11</td>
</tr>
<tr>
<td>8</td>
<td>$a_{0}, a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}$</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>$a_{0}, a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}, a_{7}$</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>$a_{0}, a_{1}, a_{2}, a_{3}, a_{4}, a_{5}, a_{6}, a_{7}, a_{8}$</td>
<td>17</td>
</tr>
</tbody>
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It is emphasized that the above filter does not seriously affect the operational cost of the simulation.

3. The convergent treatment of arbitrarily-aligned material interfaces

The effects of staircasing and the lack of properly enforced jump-conditions on both sides of arbitrarily-embedded general material interfaces, with different phase velocities and wavelengths, have negative consequences on the stability and convergence of the FDTD method. More specifically, electromagnetic fields encounter a discontinuity at these boundaries and despite the smooth solutions in each homogeneous region the final simulation is severely contaminated. In this context, we maintain the same lattice as in the second-order FDTD approach and systematically detect the path of the interface. Consider the curvilinear material boundary of Fig. 2, with $\mathbf{n} = (n_{x}, n_{y}, n_{z})$ a normal unit vector. For each of the two areas $(\text{mat} = A, B)$, a specific spatial value is defined, once the resolution has been decided. Thus, $E_{\text{mat}}^{\text{norm}}$ and $H_{\text{mat}}^{\text{norm}}$ vectors can be related to materials $E_{\text{mat}}^{\text{norm}}$ and $H_{\text{mat}}^{\text{norm}}$ via the suitable tangential/normal continuity conditions. Here, the set of
The essential material term in (19) receives the following correction: 

\[ \frac{\partial w_{hu}}{\partial whu} \]

The covariant components are utilized, while the correction of the hard-to-model \( \partial w_{hu} \) at the interface gives

\[
\frac{\partial w_{hu}}{\partial whu} = \frac{2}{\Delta \phi_{ijk}} (\hat{h}_{nu} \delta_{nu} + \hat{h}_{nu} \delta_{nu} + \hat{h}_{nu} \delta_{nu})
\]

with

\[
\delta_{nu} = 0.5 - \beta \nu_{ijk}
\]

The essential material term in (19) receives the following calculation

\[
h_n^{A+1/2} = h_n^{A+1/2} + \hat{h}_n(\mu - \delta n)\]

\[
\times \frac{h_n^{A+1/2} + \hat{h}_n(\mu + \delta n)\partial w_{hu}}{\partial whu} - \frac{\mu \partial n_{ijk} + \mu \partial n_{ijk}}{\partial whu}
\]

where the three ordinary variables in the nominator of (20) are recovered directly by extrapolation. So,

\[
h_n^{A+1/2} = (1 + \delta n_{ijk}) h_n^{A+1/2} + \delta n_{ijk} h_n^{A+1/2}
\]

\[
h_n^{A+1/2} = \frac{1}{2} (\hat{h}_n^{A+1/2} + \hat{h}_n^{A+1/2})
\]

\[
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\]

with analogous expressions holding for \( c \) components. Evaluation of the tilded variables in (22), (23) gives

\[
\hat{h}_n^{A+1/2} = \hat{h}_n^{A+1/2} + \hat{h}_n^{A+1/2}
\]

The application of (19)–(25) necessitates a predictor-corrector procedure in which the HO non-standard FDTD algorithm, serving as the predictor stage, is engaged to solve Maxwell’s equations in the entire domain, while a corrector stage adjusts the solutions locally. As a consequence, an impressive accuracy improvement is obtained along with the efficient handling of internal curvilinear interfaces.

4. Numerical results

The novel method combined with the convergent multi-space treatment of dissimilar material interfaces is validated by various practical 3-D curvilinear EMC applications and problems, which address highly deformed lattices. Towards this aim, we compare our HO non-standard FDTD outcomes with analytical solutions or measured data using very coarse grid resolutions.

Let us, first, study a 3-D problem which deals with the propagation of a Gaussian pulse inside an inhomogeneous, lossless domain discretized with a highly distorted mesh, as shown in Fig. 3 (cross-section). An indicative formula for the generation of the \( u \) grid lines (likewise for the other axes) is given below

\[
u_{i,j,k}(\tau) = \nu_0 + \Delta \nu \left( \frac{i - 1}{4} + \sin 2\pi \nu \tau \right)
\]

Fig. 3. A highly distorted FDTD grid with an adequate number of localized discontinuities.
Fig. 4. Normalized phase velocity as a function of frequency for various second-order and HO non-standard FDTD implementations. All simulations are performed on a highly distorted mesh comprising two dissimilar materials separated by an arbitrarily-aligned interface.

with $u_0 = 1.0$ and $\omega \tau = \frac{1}{4}$. The computational space consists of two materials with quite different constitutive parameters ($\varepsilon_A = 12.5\varepsilon_0$, $\mu_A = 2.45\mu_0$, $\varepsilon_B = 1.4\varepsilon_0$ and $\mu_B = 1.8\mu_0$), while their interface does not align with any of the grid lines. As can be observed such an application incorporates several interesting issues that the ordinary FDTD method can not easily confront. The excitation is placed in the area of material A, three cells away from the interface. For the truncation of the domain, we select an 8-cell PML whose absorption rates are carefully selected to absorb all traveling modes. Fig. 4 provides the normalized phase velocity versus frequency for several HO non-standard and second-order FDTD configurations. Results demonstrate that the proposed algorithm is very accurate, since its dispersion relation is far more superior to the Yee’s one. In fact, both HO implementations are remarkably close to the ideal limit of unity. Similarly, Fig. 5a and Fig. 5b show the propagation of $E_w$ and $H_v$ field components as a function of time for a prolonged simulation. Notice the stable profile and the precise modeling capabilities of our technique as compared to the reference solution [1].

We, now, continue with the analysis of some contemporary EMC structures. Herein, we explore wave penetration in an elliptical cavity with multiple rectangular inclined slots placed in a conducting plane, as shown in Fig. 6. The simulation becomes more demanding because of the cavity’s elliptical cross-section. Our configuration has the following dimensions: $a = 10\text{ cm}$, $b = 5\text{ cm}$, $L = 20\text{ cm}$, $s_1 = 1\text{ cm}$, $s_2 = 3\text{ cm}$, $h_1 = 1.7\text{ cm}$, $h_2 = 4.5\text{ cm}$ and $w = 0.2\text{ cm}$. Fig. 7 gives the shielding effectiveness of the structure versus the number of horizontal slots. Both HO and second-order FDTD computations are performed – using a 6-cell PML – while as a reference solution a modified version of [13] has been developed. As can be easily derived, the proposed methodology is far better than the ordinary Yee’s scheme despite the finer mesh of the latter. Notice that the discrepancy between the two algorithms is 7–8 dB, an issue that proves the inadequacy of conventional techniques to discern neighboring frequencies. Preserving the prior arrangement, we also study inclined slots. Three cases are examined, i.e. $\theta = 30^\circ$, $45^\circ$, $60^\circ$ and results are shown in Fig. 8. Again, HO forms offer very sufficient predictions, without producing any instabilities, even at 300 MHz.

![Fig. 6. An elliptical cavity with multiple inclined thin apertures located on a conducting plane.](image-url)
Fig. 7. Shielding efficiency versus the number of horizontal slots located on the conducting plane of an elliptical cavity for a second-order and a HO non-standard FDTD implementation.

Fig. 8. Shielding efficiency versus the number of inclined slots ($\theta = 30^\circ, 45^\circ, 60^\circ$) on the conducting plane of an elliptical cavity for a second-order and a HO non-standard FDTD realization.

As a last example, a general printed circuit board of a digital EMC circuit, described in Fig. 9, is investigated. The integrated circuits, mounted on the first layer, consist of the same material which has the following properties: $\varepsilon_A = 2.5\varepsilon_0$, $\mu_A = 1.76\mu_0$ and $\sigma_A = 0.22$ S/m, while those of the substrate are $\varepsilon_B = 4.7\varepsilon_0$, $\mu_B = \mu_0$ and $\sigma_B = 0.03$ S/m. It is evident that the material interfaces, so created, require a careful modeling, especially in the specific application which considers losses and involves decoupling capacitances of 0.01 $\mu$F that give the return pass of high frequency energy. The dimensions are $b_1 = 18.65$ mm, $b_2 = 16.93$ mm and $b_3 = 2.5$ mm, while the domain comprises $36 \times 24 \times 8$ cells with $\delta x = \delta y = \delta z = 1.5$ mm and $\delta t = 25.32$ ps. Fig. 10a and b demonstrate the variation of $S_{11}$ and $S_{21}$ parameter as a function of frequency for a second-order and a HO FDTD realization. The reference solution has been obtained from [14].

As can be deduced, the HO outcomes exhibit a notable accuracy, without any discrepancies in the frequency spectrum. Conversely, the Yee’s algorithm is proven to be insufficient – especially in the detection of acute peaks – despite the much finer $82 \times 64 \times 156$ lattice it employs. Thus, the proposed method attains a virtually 85% reduction of the CPU time and memory requirements. Finally, Fig. 11 indicates the normalized phase velocity, measured at a prefixed point, for different discretizations. Results confirm that all non-standard simulations perform much better than the second-order ones, which deviate significantly from acceptable values as resolution decreases. This is a very instructive deduction especially in the analysis of large-scale structures where traditional techniques lack to provide adequate solutions.
5. Conclusions

The development of an accurate HO non-standard FDTD method for the advanced analysis of complicated EMC systems and the consistent processing of general material interfaces in deformed meshes in non-orthogonal curvilinear lattices is introduced in this paper. The key premise stems from the flux-based field formulation via an efficient HO covariant and contravariant strategy combined with a low-pass filtering algorithm. Hence, the fully non-orthogonal process launches additional degrees of freedom appropriate for various sets of dissimilar media. To treat late-time instabilities, enhanced leapfrog integration is established, whereas a family of self-adaptive compact operators copes with the widened spatial stencils. For unbounded problems, the proposed HO concepts are applied to the construction of curvilinear PMLs, which offer a strong annihilation of outgoing waves. Numerical results prove that the proposed method is highly accurate, notably stable and requires sufficiently coarser grid resolutions, thus diminishing the overall burden.

References