An Unconditionally Stable Higher-Order ADI-FDTD Technique for the Dispersionless Analysis of Generalized 3-D EMC Structures

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Abstract—An enhanced higher-order 3-D ADI-FDTD algorithm for the accurate and unconditionally stable modeling of complex curvilinear EMC problems, is introduced in this paper. The new technique launches a topologically-consistent family of non-standard concepts which eliminate the serious dispersion errors of the usual ADI-FDTD scheme as time-step increases, and cancels its strong dependence on cell shape or mesh resolution. Thus, temporal increments can greatly exceed the Courant limit with superior stability and convergence levels. To optimize computing, higher-order curvilinear PML absorbers are also developed. Theoretical analysis, along with the numerical verification of diverse structures reveal that the proposed method is highly precise, subdues the vector-parasitic mechanisms of the ADI approach and achieves significant computational savings.

INTRODUCTION

The gradually increasing complexity of modern microwave devices has stipulated the needs for numerical schemes that combine top performance with realistic computational costs. A restrictive factor in the finite-difference time-domain (FDTD) analysis of such applications, where cells are much smaller than the shortest wavelength, is the Courant limit enforcing excessive numbers of temporal iterations to reach steady state. Lately, an efficient unconditionally stable alternating-direction implicit (ADI) formulation for the FDTD method has been developed [1, 2]. Its time-steps are not confined by cell attributes, while the profile of the modeled waveforms is highly resolved. However, various instructive studies [3-5] and algorithms [6-9] have shown that large dispersion errors are induced as the time interval is increased yielding thus, incorrect simulations.

In this paper, a 3-D ADI-FDTD technique, based on curvilinear tensorial forms, is presented for the mitigation of the preceding lattice reflection constraints and the consistent analysis of electromagnetic compatibility (EMC) applications. Through an advanced discretization policy, the novel algorithm introduces a parametric set of accurate higher-order non-standard schemes and conducts alternations in respect to mixed coordinates rather than to each direction. This procedure suppresses the inherent dispersion errors and allows time-steps to substantially surpass the Courant criterion. Also, the overall solution is further enhanced via higher-order perfectly matched layers (PMLs) which are derived in terms of a fully curvilinear procedure. The merits of the proposed methodology are numerically certified with several difficult arbitrarily-curved arrangements that would otherwise necessitate elongated simulations.

THE ADVANCED NON-STANDARD TOPOLOGICAL FORMULATION

The key feature of the higher-order (HO) strategy is the new class of non-standard operators which eliminate the structural defects of the common ADI-FDTD method. Their form is

\[ P_{\xi,\zeta}^{m} f_{n,\zeta} = \frac{g_{\nu,\gamma,\nu}}{c_{\nu}(kL\Delta\zeta)} \sum_{m=1}^{M} Q_{m}^{\zeta} \left( \sum_{l=1}^{L} P_{\nu,\nu}^{m} W_{\nu,\nu}^{(m)} f_{n,\zeta} \right), \tag{1} \]

where \( M \) is the order of accuracy and \( \zeta \) a variable of the curvilinear coordinate system \((\nu,\gamma,\nu)\) defined by a set of \( g \) metrics. Coefficient \( L \) is very important, since it defines the suitable set of stencils, \( \Delta\gamma \), along each axis. For lattice consistency, operators \( W_{\nu,\nu}^{(m)} \), cover all optimal node tessellations – independent of cell shape or mesh resolution – hence enabling the use of fairly coarse grids and multi-directional modeling of structural peculiarities. A typical choice of the prior parameters is \( M = 4 \) and \( L = 3 \), while an indicative \( \nu \)-directed \( W_{\nu,\nu}^{(m)} \) is given by

\[ W_{\nu,\nu}^{(m)} f_{n,\zeta} = \frac{m^{2}}{8(2m-1)} \left( f_{n,\zeta} - m f_{n+2\xi,\zeta} - m f_{n-2\xi,\zeta} \right), \tag{2} \]

in which only the respective spatial increments towards \( \nu,\gamma,\nu \) are depicted. Function \( c_{\nu} \), in (1), controls the convergence of the method, whereas the degrees of freedom \( p_{\nu,\nu}^{m}, Q_{m}^{\zeta} \) augment its error-annihilating behavior via the fulfillment of

\[ \sum_{l=1}^{L} p_{\nu,\nu}^{m} = 1 \quad \forall m, \quad \sum_{m=1}^{M} Q_{m}^{\zeta} = 1/2. \tag{3} \]

Therefore, space and time derivatives are approximated by very precise HO schemes and receive the following notation

\[ D_{\xi} f_{n,\zeta} = \frac{q_{\lambda}}{4\Delta\lambda} \left( f_{n+1,\zeta} + f_{n-1,\zeta} + f_{n,\zeta+1} + f_{n,\zeta-1} - 4f_{n,\zeta} \right), \tag{4} \]

\[ T_{\xi} f_{n,\zeta} = \left( f_{n+1/2,\zeta} - f_{n-1/2,\zeta} \right) / c_{\nu} \Delta\lambda - q_{b} \partial_{\nu} f_{n,\zeta}, \tag{5} \]

with \( q_{\lambda}, q_{b} \) being certain tuning factors and \( c_{\nu} \) the non-standard function of \( \Delta\zeta \). The summation, in (4), combined with a self-adaptive compact regime, treats boundaries via the \( \pm \) signs.

THE 3-D HO CURVILINEAR ADI-FDTD TECHNIQUE

The generalized HO classification hosts a dispersionless ADI-FDTD scheme which, unlike Yee’s single advance from the \( n \)-th to \((n+1)\)-th time-step, involves two sub-iterations: one from \( n \) to \( n+1/2 \) and the other from \( n+1/2 \) to \( n+1 \). Let us con-
consider Ampere’s law expressed in its covariant form as \(\varepsilon_0 \varepsilon_u + \mu_0 \sigma u = \gamma_{\sigma u} \partial_t \mu - \gamma_{\sigma u} \partial_t \varepsilon_u\) via the metrical coefficients \(g\). In the first sub-iteration, partial derivative \(\partial_t \mu\) is implicitly described by its unknown pivotal values at \(n+1/2\), while \(\partial_t \varepsilon_u\) is explicitly substituted by its already computed values at \(n\). Consequently,

\[
A^* e_{\rho pos}^{n+1/2} - A e_{\rho pos}^{n\rho} = g_{\omega m} D_w \left[ h_u^{n+1/2} \right] - g_{\omega m} D_w \left[ h_u^{n\rho} \right],
\]

with \(\rho pos = (i+1/2, j, k)\) and \(A^2 = 2(\alpha + \gamma_{\sigma u}/\Delta t)\) including additionally third-order time differentiations in \(g_{\omega m}\). The same procedure for the magnetic \(h_u\) quantities (Faraday’s law), results in

\[
\begin{align*}
B^* h_u^{n+1/2} - B h_u^{n\rho} = g_{\omega m} D_v e^{n+1/2} - g_{\omega m} D_v e^{n\rho},
\end{align*}
\]

Now, \(\rho\) = \((i+j+1/2, \theta)\), \(B^2 = 2(g_{\omega m}/\Delta t)\). Due to the concurrent definition of \(e_{\rho\rho}\), \(h_u^{n\rho}\), in (6), we replace \(h_u^{n\rho}\) via (7) and

\[
egin{align*}
& a_1 e^{n+1/2}_{\rho\rho} - a_2 e^{n+1/2}_{\rho\rho,j+1/2,k} - a_3 e^{n+1/2}_{\rho\rho,i,j-1/2,k} = \\
& a_1 e^{n+1/2}_{\rho\rho,j+1/2,k} + h_u D_v e^{n+1/2}_{\rho\rho,i,j+1/2,k} - h_u D_v e^{n+1/2}_{\rho\rho,i,j-1/2,k} = \\
& -h_u \left[ D_v \left( e^{n+1/2}_{\rho\rho,j+1/2,k} + D_v \left( e^{n+1/2}_{\rho\rho,i,j-1/2,k} \right) \right) \right],
\end{align*}
\]

where \(a_{\rho\rho}, b_{\rho\rho}\) are suitable system metrics. (8) have a tridiagonal form for every \(j\), they can be recursively solved with a trivial overhead. On the other hand, the second sub-iteration reverses the time-update of \(\partial_t \mu\) and \(\partial_t \varepsilon_u\) to give similar notions. Application to all Maxwell equations yields the full set of HO curvilinear formulae that evaluate fields along alternating directions. Expressing (8) in matrix notation, one can readily obtain

\[
\begin{align*}
\Xi_\rho^{n+1/2} = \Theta_\rho^{n} \Xi_\rho^{n} & \Rightarrow \Xi_\rho^{n+1/2} = \left[ \Xi_\rho, \Theta_\rho, \Xi_\rho \right]^{n} \Xi_\rho^{n+1/2},
\end{align*}
\]

with \(\Xi_\rho = [E^\rho, H^\rho]\) and the sparse \(\Theta_\rho^{n}\) acquired by the non-standard time-marching concepts of (8). Theoretical analysis – provided in the full paper – indicates that all eigenvalues in (9) are always less than or equal to 1 and therefore the HO method is unconditionally stable. Its new dispersion relation becomes

\[
\sin(\alpha \Delta z) = \frac{26(\Delta t)^{1/4}}{4^{n+1/2} (\mu_{\varepsilon} \varepsilon_0) \frac{1}{2}} F \left( S_{\varepsilon}^2, S_{\mu}^2, S_{\mu}^2, \mu_{\varepsilon} \right),
\]

as a function of the existing ADI-FDTD relation, \(F\), with \(S_{\varepsilon} = (\Delta t/\Delta z) \sin(k_{\Delta z}^\rho/2)\). Clearly, (10) exhibits a notable superiority.

**Numerical Results**

To verify the proposed algorithm, several 3-D scattering and EMC curvilinear problems, are analyzed with a resolution of \(\lambda/6\) (instead of Yee’s \(\lambda/170\) one) and truncated by HO PMLs. Fig. 2 gives the numerically-calculated phase velocity for various CFLN = \(\Delta t_{ADI}^{\mu} / \Delta t_{FDTD}^{\mu}\) values due to a dielectric sphere (\(\varepsilon = 5.5\varepsilon_0\)). As observed the HO ADI-FDTD method outperforms its 2nd-order counterpart. Next, we consider the hard-to-model cavity of Fig. 1a with \(a_1 = 23.44\ mm, a_2 = 11.96\ mm, a_3 = 32.87\ mm, b_1 = 6.28\ mm, b_2 = 10.26\ mm, b_3 = 19.34\ mm\) and \(\theta = 45^\circ\). Results for the S21 parameter in Fig. 3a and the first five resonances in Table I (CFLN = 7.215) prove the high precision and savings of our technique (almost 85% grid and CPU time reduction). Finally, Fig. 3b shows the normalized phase velocity (large temporal increments) for the aperture of Fig. 1b. Note the great discrepancies of the regular ADI-FDTD approach.

**H.O. ADI-FDTD method**

A practically dispersionless ADI-FDTD method, based on systematic HO non-standard forms, for 3-D curvilinear EMC problems has been presented in this paper. The technique decreases radically the overall burden, while demonstrating an overwhelming accuracy even for considerably large time-steps.

**References**


