The propagation of the electromagnetic field inside a conducting slab extended in the infinity is treated by the use of a boundary element technique. The excitation is a current source arbitrarily oriented inside the conducting material. The conducting slab is situated in the air. The magnetic field intensity is expressed in terms of the electric vector potential $T$ and the magnetic scalar potential $\phi$. The integral equations for these two field quantities are formed by the use of the Green's function method into the original Helmholtz and Laplace equations for $T$ and $\phi$, respectively. Apart from the boundary conditions, the need of imposing at least one more additional condition between the field quantities becomes obvious during the solution procedure, with the help of a boundary element method. Examples are given using two materials with quite different electric properties, aluminum and graphite. The work can be extended in more complicated stratified media to face propagation problems in geometries with complex boundaries.

II. SOLUTION PROCEDURE

A current dipole is situated in a distance $h$ from the upper surface of the slab and diffuses a current density as follows:

$$
\mathbf{J}_e = \mathbf{J}_o \delta(r) e^{i\omega t} \hat{r},
$$

$$
\mathbf{J}_m = \mathbf{J}_o \delta(r) \sin \theta \cos \phi \hat{x}_0 + \sin \theta \sin \phi \hat{y}_0 + \cos \theta \hat{z}_0,
$$

where $\theta, \phi$ are the angular polar coordinates, which determine the direction of the dipole, $\delta(r)$ is the Dirac's delta function and $t$ is the position vector of the source.

Starting from the definition of the quantities $T$ and $\phi$, $H = \mathbf{T} - \nabla \phi$, the family of equations which describe the problem are

$$
\nabla^2 T - k_0^2 T = -\nabla \times \mathbf{J}_e,
$$

$$
\nabla \phi - k_0^2 \phi = 0,
$$

plus the gauge condition

$$
\nabla \cdot \mathbf{T} - k_0^2 \phi = 0,
$$

where $m = 1$ for the conducting material, with $k_1 = j \omega \sigma - \omega^2 \mu \varepsilon$ and $m = 0$ with $k_0 = -\omega^2 \mu \varepsilon$ for the air.

Apart from all the others, the use of the $T - \phi$ method provides the flexibility of having zero values for the electric vector potential $T$ in the air.

The next step is to transform Eqs. (2) and (3) into integral forms. This may be succeeded by the use of the Green's function method. Finally we obtain

$$
T_m(x,y,z) = C_1 \int_V G_m(\nabla \mathbf{J}_e) \cdot dV - C_2 \int_S \left( T_m \frac{\partial G_m}{\partial n} \right) dS,
$$

$$
\phi_m(x,y,z) = -C_1 \int_S \left( \phi_m \frac{\partial G_m}{\partial n} - G_m \frac{\partial \phi_m}{\partial n} \right) dS,
$$

where $G_1 = e^{-kr}/(4\pi r)$ and $G_0 = 1/(4\pi r)$ are the Green's function of the air for the Helmholtzian and Laplacian operators respectively, $m = 0, 1$ and $j = x, y, z$. $C_i = 0$ for non conducting materials, $C_1 = C_2 = 1$ for field points inside the volume $V$, and $C_1 = C_2 = 2$ for field points belonging to the surface $S$. $n$ is the outward normal to the dividing surfaces unit vector.

The solution of Eqs. (5) and (6) may be achieved by the use of a boundary element technique. The upper and
lower boundary surface are divided into \( m^+ \) and \( m^- \) elements, respectively. In every element, equations similar to Eqs. (5) and (6) are written. It is obvious that the coefficients \( C_1 \) and \( C_2 \) are equal to 2 for the upper boundary elements and equal to \(-2\) for the boundary elements of the lower surface, in which the outward normal to the surface unit vector has the opposite direction of the same vector in the upper surface. Finally we have

\[
T_{mjl}(x,y,z) = 2 \int_y G_m(\nabla x J_i) \, dV \\
-2 \sum_{l=1}^{m^+} \left( T_{mjl} \frac{\partial G_{mll}}{\partial z} - G_{mll} \frac{\partial T_{mjl}}{\partial z} \right) \Delta S_i \\
+2 \sum_{j=1}^{m^-} \left( T_{mjl} \frac{\partial G_{mfl}}{\partial z} - G_{mfl} \frac{\partial T_{mjl}}{\partial z} \right) \Delta S_f,
\]

(7)

where the signs \(+\) and \(-\) represent the values in the upper and lower boundary, and \( \Delta S_i \) is the area of the element \( i \).

It is easy to prove that the unknown quantities in every boundary element are less than one of the available equations. At this point the need for one more additional condition is born.

The boundary condition for the continuity of the tangential components of the magnetic field intensity give

\[
T_{1x} = \frac{\partial \Omega_{0x}}{\partial x}, \\
T_{1y} = \frac{\partial \Omega_{0y}}{\partial y}.
\]

(9)

(10)

Taking as an additional condition that the \( \Omega \) is passing continuously through boundaries, the above equations give zero values for \( T_x \) and \( T_y \) everywhere in the boundaries, and Eq. (7) takes the form

\[
0 = 2 \int G_m(\nabla x J_i) \, dV + 2 \sum_{l=1}^{m^+} G_{1il} \frac{\partial T_{1il}}{\partial z} \Delta S_i \\
-2 \sum_{j=1}^{m^-} G_{1ij} \frac{\partial T_{1ij}}{\partial z} \Delta S_f, \quad i=x,y.
\]

(11)

Equation (11) gives two independent systems of equations, from which the quantities \( \frac{\partial T_{1x}}{\partial z} \) and \( \frac{\partial T_{1y}}{\partial y} \) are calculated.
$$\Omega_{tl} = -2 \sum_{i \neq 1}^{m^+} \left( \Omega_{li}^+ \frac{\partial G_{ill}}{\partial z} + G_{ill} \frac{\partial \Omega_{li}^+}{\partial z} \right) \Delta S_i,$$

$$+ 2 \sum_{f \neq 1}^{m^-} \left( \Omega_{lf}^+ \frac{\partial G_{llf}}{\partial z} - G_{llf} \frac{\partial \Omega_{lf}^+}{\partial z} \right) \Delta S_f, \quad (15)$$

$$\Omega_{0l} = \Omega_{l1}^+ = 2 \sum_{i \neq 1}^{m^+} \left[ \Omega_{li}^+ \frac{\partial G_{0ll}}{\partial z} + G_{0ll} \left( T_{1zi}^+ - \frac{\partial \Omega_{li}^+}{\partial z} \right) \right] \Delta S_i,$$

$$- 2 \sum_{f \neq 1}^{m^-} \left[ \Omega_{lf}^+ \frac{\partial G_{0lf}}{\partial z} - G_{0lf} \left( T_{1zf}^- - \frac{\partial \Omega_{lf}^+}{\partial z} \right) \right] \Delta S_f. \quad (16)$$

$$\Omega_{0l}^+ = \Omega_{l1}^-$$

$$= - 2 \sum_{f \neq 1}^{m^-} \left[ \Omega_{lf}^+ \frac{\partial G_{0lf}}{\partial z} - G_{0lf} \left( T_{1zf}^- - \frac{\partial \Omega_{lf}^+}{\partial z} \right) \right] \Delta S_f. \quad (17)$$

After the calculation of the values of the unknown in the two surfaces, the field inside the conducting material can be calculated with the help of Eqs. (5) and (6).

**III. NUMERICAL EXAMPLES—CONCLUSIONS**

Two slabs, one from aluminum, with conductivity of $7 \times 10^4$ S/m, and one from graphite, with conductivity of $7 \times 10^4$ S/m, are used. The frequency is 50 Hz, the moment of the dipole 1 A m and the orientation of the dipole is taken perpendicular to the surfaces and towards $z$ direction. Because of the type of the orientation, there is a symmetry of the field in respect to the $z$ axis (see Figs. 2-5).

The method described so far provides a useful tool for the evaluation of the propagation phenomenon in stratified media. The example of the conducting slab shows the ability of the method to face a three-dimensional problem as a two-dimensional one. The choice of the appropriate additional assumption is of great importance, and that is the point which may make the solution of the problem quite easy. Of course this assumption may not influence the boundary conditions and the definition of the related quantities.